

SURFACE AREAS & VOLUMES

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INTRODUCTION

We are familiar with many geometrical figures around us in our daily life. Some of them are plane figures such as circle, polygons *etc.* We also come across many solid geometrical shapes such as cuboids, cones, cylinders, spheres and hemispheres *etc.* We have in some details learnt in our earlier classes about how to calculate the areas, perimeters of the plane figures. Also we have learnt how to calculate the surface areas and volumes of these solid bodies. In our current discussion, we shall extend our exploration to those solids which are made up of combinations of two or more such solids, for example a conical circus tent with cylindrical base is a combination of a right circular cylinder and a right circular cone, an ice-cream cone which is a combination of a cone and a hemisphere *etc.* We shall also evaluate the areas and volumes of solids such as buckets (best known as *frustum of a cone*) *etc.*

★ SOME IMPORTANT FORMULAE

01. Cuboid:

$$\text{Lateral surface area} = 2(l + b).h$$

$$\text{Total surface area} = 2(lb + bh + hl)$$

$$\text{Volume} = lbh$$

02. Cube:

$$\text{Lateral surface area} = 4a^2$$

$$\text{Total surface area} = 6a^2$$

$$\text{Volume} = a^3$$

03. Sphere:

$$\text{Surface area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

04. Hemisphere:

$$\text{Curved surface area} = 2\pi r^2$$

$$\text{Total surface area} = 3\pi r^2$$

$$\text{Volume} = \frac{2}{3}\pi r^3$$

05. Cylinder:

$$\text{Curved surface area} = 2\pi rh$$

$$\text{Total surface area} = 2\pi r^2 + 2\pi rh$$

$$\text{Volume} = \pi r^2 h$$

06. Cone:

$$\text{Curved surface area} = \pi rl, \text{ where } l^2 = r^2 + h^2$$

$$\text{Total surface area} = \pi r^2 + \pi rl$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

07. Frustum of a cone:

$$\text{Curved surface area} = \pi l(R + r), \text{ where } l = \sqrt{h^2 + (R - r)^2}$$

$$\text{Total surface area} = \pi l(R + r) + \pi(R^2 + r^2), \text{ where } l = \sqrt{h^2 + (R - r)^2}$$

$$\text{Volume} = \frac{1}{3}\pi h(R^2 + r^2 + rR)$$

EXERCISE FOR PRACTICE

Q01. Fill in the blanks:

- The total surface area of a cuboid of dimensions $a \times a \times b$ is _____.
- The volume of right circular cylinder of base radius r and height $2r$ is _____.
- If the height of a cone is same as its diameter of the base, its volume is _____.

(d) The lateral surface area of a hollow cylinder of outer radius R, inner radius r and height h is given as _____.

(e) If radius of a sphere is doubled, its volume becomes _____ times the volume of original sphere.

(f) If the radius of a sphere is halved, its volume becomes _____ times the volume of initial sphere.

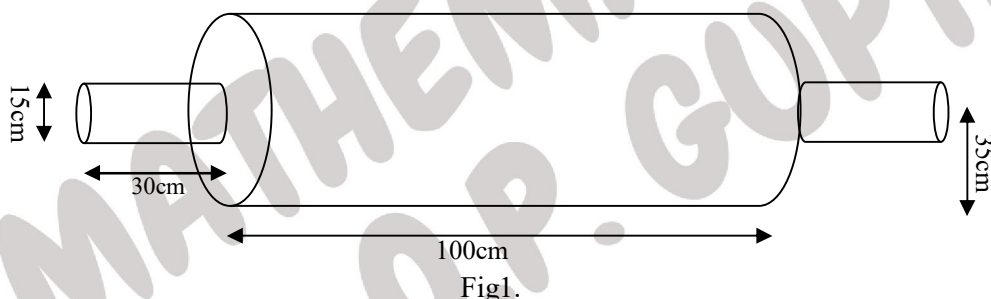
$$\left[\text{Ans. (a) } 2a(a+2b) \text{ (b) } 2\pi r^3 \text{ (c) } \frac{2}{3}\pi r^3 \text{ (d) } 2\pi h(R+r) \text{ (e) } 8 \text{ (f) } \frac{1}{8} \right]$$

Q02. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane. [Ans.1:2]

Q03. A spherical copper shell of external diameter 18 cm is melted and recaste into a solid cone of base radius 14 cm and height $4\frac{3}{7}$ cm . Find the inner diameter of the shell. [Ans.16 cm]

Q04. From a solid cylinder of height 24 cm and diameter 10 cm, two conical cavities of same radius as that of the cylinder are hollowed out. If the height of each conical cavity is half the height of cylinder, find the total surface area of remaining portion of cylinder. [Ans.1162.85 cm²]

Q05. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. [Ans.572 cm²]



Q06. A roller pin is made by joining 3 cylindrical pieces of wood as shown in the Fig1. Find the cost of painting it at the rate of 10 paise per sq. cm. [Ans. ₹ 3525.86]

Q07. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank to her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled? [Ans.100 min]

Q08. A toy is in the form of a cone on a hemi-sphere of diameter 7 cm. The total height of the toy is 14.5 cm. Find the volume and total surface area of toy. [Ans.231 cm³, 203.9 cm²]

Q09. A vessel in the form of hemi-sphere is mounted by a hollow cylinder. The diameter of the bowl is 14 cm and the total height of the vessel is 13 cm. Find its capacity. [Ans.1642.66 cm³]

Q10. A cylindrical container with radius and height 4 cm and 8 cm respectively is filled with ice-cream and the ice-cream is distributed to 8 children in equal cone having hemi-spherical tops. If the height of the conical portion is 4 times the radius of its base, find the radius of the ice-cream cone. [Ans.2 cm]

- Q11.** A tent has cylindrical shape surmounted by a conical roof. The radius of the cylindrical base is 20 m. The total height of the tent is 6.3 m and height of the cylindrical portion is 4.2 m. Find the volume and surface area of the tent. [Ans. $616 \text{ cm}^3, 1792.68 \text{ cm}^2$]
- Q12.** A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of hemisphere is known to be 14 cm and the total height of vessel is 13 cm. Find the inner surface area of the vessel. [Ans. 572 cm^2]
- Q13.** A conical vessel with internal radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. Find the volume of water overflows. [Ans. 188.57 cm^3]
- Q14.** A given amount of wax in cylindrical form is heated in a metal container and then poured into another container. A new candle is formed which is shaped like a fish. Fill in the blanks:
(a) The volume of the wax _____ (is changed/ remains the same).
(b) The total surface area _____ (is changed/ remains the same).
[Ans. (a) remains the same (b) is changed]
- Q15.** How many silver coins, 1.75 cm in diameter and of thickness 2mm, must be melted to form a cuboid of dimensions $5.5 \times 10 \times 3.5 \text{ cm}^3$? [Ans. 400]
- Q16.** A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is known to be 10 cm and its base is of radius 3.5 cm, find the total surface area of the article. [Ans. 374 cm^2]
- Q17.** A sphere of radius 3 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 6 cm. If the sphere is submerged completely, by how much will the surface of water be raised? [Ans. 1 cm]
- Q18.** The radii of the ends of the frustum of a cone are 14 cm and 21 cm with the slant height being 8 cm. Find the area of its curved surface. [Ans. 880 cm^2]
- Q19.** Find the volume of a frustum of a cone 4 m high whose face radii are 7 m and 14 m. [Ans. 1437.33 cm^3]
- Q20.** A lamp shade made of a special chart paper is in the form of a frustum of a cone open at both ends. The radii of its ends are 16 cm and 24 cm and its height is 6 cm. Find the cost of paper used, if 1 cm^2 costs ₹0.70. [Ans. ₹880]
- Q21.** A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder. [Ans. 1 cm]
- Q22.** A vessel is in the form of an inverted cone. Its height is 8 cm and radius of its top, which is open, is 5 cm. It is filled with some liquid up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm, are dropped into the vessel, one-fourth of the liquid flows out. Find the number of lead shots dropped. [Ans. 100]
- Q23.** A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to the base. If the frustum so obtained is melted and drawn into a wire of diameter 1 cm, find the length of the wire. [Ans. 3111.11 cm]

- Q24.** An open container is in form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at a rate of ₹20 per liter. Also find cost of material used to make the container if it costs ₹8 per 100 cm². [Ans. ₹ 209, ₹156.75]
- Q25.** A container shaped like a circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 6 cm and diameter 3 cm having a hemispherical shape on the top of the same radius of base as that of cone. Find the number of such cones that can be filled with the ice cream. [Ans.80]
- Q26.** A cone of radius 10 cm is divided into two parts by drawing a plane through the midpoint of its axis, parallel to its base. Compute the ratio of the volume of two parts.
- Q27.** Two spheres of the same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the bigger sphere.

📖 If you've any doubt or want help, please post the image (screenshot) of your question in the Telegram Group <https://t.me/Mathematicia4Tenth>

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We have taken utmost care while preparing this draft. Still chances of human error can't be ruled out.

Please inform us about any Typing error / mistake in this document.

This will help many future learners of Mathematics.

We express our gratitude to all the teachers of our MATHEMATICIA Groups, especially Sachin Pandey (St Marys' School, Rudra Pur) & Maneet Singh (Mind Evolution Academy, Tilak Nagar).

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